DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS SERIES B
Volume **17**, Number **5**, July **2012**

doi:10.3934/dcdsb.2012.17.1575

pp. **1575–1584**

IMPACT OF α-STABLE LÉVY NOISE ON THE STOMMEL MODEL FOR THE THERMOHALINE CIRCULATION

Xiangjun Wang and Jianghui Wen
School of Mathematics and Statistics
Huazhong University of Science and Technology
Wuhan, 430074, China

Jianping Li
State Key Laboratory of Atmospheric Sciences and Geophysical Fluid Dynamics
Institute of Atmospheric Physics, Chinese Academy of Sciences
Beijing, 100029, China

Jinqiao Duan
Department of Applied Mathematics, Illinois Institute of Technology
Chicago, IL 60616, USA

Abstract. The Thermohaline Circulation, which plays a crucial role in the global climate, is a cycle of deep ocean due to the change in salinity and temperature (i.e., density). The effects of non-Gaussian noise on the Stommel box model for the Thermohaline Circulation are considered. The noise is represented by a non-Gaussian α-stable Lévy motion with $0 < \alpha < 2$. The α value may be regarded as the index of non-Gaussianity. When $\alpha = 2$, the α-stable Lévy motion becomes the usual (Gaussian) Brownian motion.

Dynamical features of this stochastic model is examined by computing the mean exit time for various α values. The mean exit time is simulated by numerically solving a deterministic differential equation with nonlocal interactions. It has been observed that some salinity difference levels remain in certain ranges for longer times than other salinity difference levels, for different α values. This indicates a lower variability for these salinity difference levels. Realizing that it is the salinity differences that drive the thermohaline circulation, this lower variability could mean a stable circulation, which may have further implications for the global climate dynamics.

1. **Introduction.**

The Thermohaline Circulation (THC), characterizing the circulation of the deep ocean driven by the marine change of temperature and salinity, plays an significant role on the global climate variability. A simple deterministic box model was proposed by Stommel [1] in 1961, to examine some features of this oceanic process [2]-[4].

However, the global oceanic system is affected by some random or uncertain processes such as stochastic environment fluctuations and random input. There are also other difficulties in including unrepresented mechanisms, uncertain observation,

2000 *Mathematics Subject Classification.* Primary: 60H10, 65C50; Secondary: 86A05.
Key words and phrases. α-stable Lévy motion, thermohaline circulation, Stommel box model, mean exit time, colored noise.

This research was partly supported by an open research grant of the State Key Laboratory of Atmospheric Sciences and Geophysical Fluid Dynamics, Chinese Academy of Sciences, and the Fundamental Research Funds for the Central Universities, HUST: 2010 ZD037.

and unresolved scales in modeling and simulation of oceanic motions. It is desirable to take stochastic or uncertain effects into account. Cessi [5] proposed a box model of stochastically-forced thermohaline processes. Griffies et. al. [6] and Lohmann et. al. [7] also considered Gaussian stochastic effects on simple models of THC. Our aim in this paper is to discuss the impacts of the non-Gaussian noise on the THC.

We will consider a simple model for THC as a scalar stochastic differential equation (SDE) in the following form

$$dy_t = f(y_t)dt + dL_t, \qquad y_0 = y, \tag{1}$$

where $f(y)$ is a deterministic vector field and L_t is a scalar non-Gaussian Lévy motion.

Recall that a scalar Lévy motion $\{L_t\}$, in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, is characterized by a drift parameter θ, a variance parameter $d > 0$ and a non-negative Borel measure ν defined on $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$ and concentrated on $\mathbb{R} \setminus \{0\}$. The so-called Lévy jump measure ν satisfies the condition

$$\int_{\mathbb{R}\setminus\{0\}} \min(z^2, 1)\, \nu(dz) < \infty. \tag{2}$$

According to the Lévy-Khintchine formula [8], the characteristic function for L_t is

$$\mathbb{E}e^{i\lambda L_t} = \exp\{i\theta\lambda t - dt\frac{\lambda^2}{2} + t\int_{\mathbb{R}\setminus\{0\}} (e^{i\lambda z} - 1 - i\lambda z I_{|z|<1})\nu(dz)\}, \tag{3}$$

where $\mathbb{E}$ is the expectation with respect to the probability measure $\mathbb{P}$ and I_S is the indicator function on the set S. The generator A_0 for this Lévy motion L_t is

$$A_0\varphi(y) = \theta\varphi'(y) + \frac{d}{2}\varphi''(y) + \int_{\mathbb{R}\setminus\{0\}} [\varphi(y+z) - \varphi(y) - I_{\{|z|<1\}} z\varphi'(y))]\nu(dz). \tag{4}$$

Thus the generator A for y_t in the stochastic differential equation (1) is

$$\begin{aligned} &A\varphi(y) \\ =&f(y)\varphi'(y) + \theta\varphi'(y) + \frac{d}{2}\varphi''(y) + \int_{\mathbb{R}\setminus\{0\}} [\varphi(y+z) - \varphi(y) - I_{\{|z|<1\}} z\varphi'(y)]\nu(dz). \end{aligned} \tag{5}$$

The rest of this paper is organized as follows. In Section 2, we present a stochastic Stommel model for characterizing some dynamical features of THC, which is a stochastic differential equation with a symmetric $\alpha-$stable Lévy motion. In Section 3, we compute the mean exit time (MET) by solving a differential-integral equation, as the parameter α varies between 0 and 2. Numerical results and discussions are described in Section 4.

2. **Stochastic Stommel model of thermohaline circulation.** A simple model for the oceanic thermohaline circulation is Stommel's box model, where the ocean is described by two boxes, a low-latitude box with temperature T_e and salinity S_e, and a high-latitude box with temperature T_p and salinity S_p. The two boxes are interacted by tubes with a flux Ψ near the surface and at depth for heat exchanges which have different volumes as shown in Figure 1.

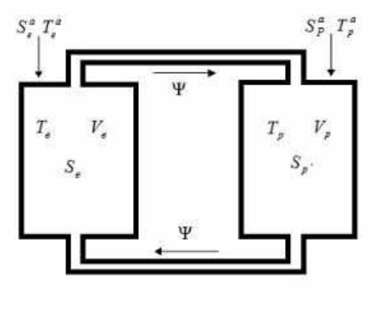

FIGURE 1. From Dijkstra ([17]) and Glendinning ([11]). Diagram of the box model. One box is for the polar region (high latitude) and the other box is for the equatorial region (low latitude).

Here we follow the presentation in [11, 12]. Define

$$\Psi = \gamma(\alpha_T(T_e - T_p) - \alpha_S(S_e - S_p)),$$

where α_T and α_S are respectively the thermal and haline expansivity coefficients. The model assumes that heat is added to the polar or equatorial box at a rate T_p^a or T_e^a respectively, from the atmosphere, and that salinity is increased at the equator with rate S_e^a and decreased at the polar with rate S_p^a. Note that the bulk exchanges between the boxes depend on the density gradient, and the dynamics of the Stommel model depends on the meridional gradient of temperature and salinity. The model equations are:

$$V_e \frac{dT_e}{dt} = C_e^T(T_e^a - T_e) + |\Psi|(T_p - T_e), \tag{6}$$

$$V_p \frac{dT_p}{dt} = C_p^T(T_p^a - T_p) + |\Psi|(T_e - T_p), \tag{7}$$

$$V_e \frac{dS_e}{dt} = C_e^S(T_e^a - S_e) + |\Psi|(S_p - S_e), \tag{8}$$

$$V_p \frac{dS_p}{dt} = C_e^S(T_p^a - S_p) + |\Psi|(S_e - S_p). \tag{9}$$

It is assumed that the relaxation rates for the temperature are equal and constant, $C_e^T/V_e = C_p^T/V_p \triangleq R_T$ and similarly, $C_e^S/V_e = C_p^S/V_p \triangleq R_S$.

The temperature difference $T = T_e - T_p$ and the salinity difference $S = S_e - S_p$ are assumed to simplify the equations which can be combined into two differential equations as follows

$$\frac{dT}{dt} = R_T[(T_e^a - T_p^a) - T] - 2|\Psi|T(\frac{1}{V_e} + \frac{1}{V_p}), \tag{10}$$

$$\frac{dS}{dt} = R_S[(S_e^a - S_p^a) - S] - 2|\Psi|S(\frac{1}{V_e} + \frac{1}{V_p}). \tag{11}$$

We set $T^a = T_e^a - T_p^a$ and $S^a = S_e^a - S_p^a$. Using the dimensionless variables $x = T/T^a$, $y = \alpha_S S/\alpha_T T^a$, and switching the time scale by $\tau = R_s t$, the equations

(10) and (11) may be rewritten as

$$\frac{dx}{d\tau} = \frac{R_T}{R_S}(1-x) - A|x-y|x, \tag{12}$$

$$\frac{dy}{d\tau} = \zeta(s) - (1 + A|x-y|)y, \tag{13}$$

in which

$$A = \frac{\gamma \alpha_T T^\alpha}{R_S}(\frac{1}{V_e} + \frac{1}{V_p})$$

and

$$\zeta = \frac{\alpha_S S^\alpha}{\alpha_T T^\alpha}.$$

Given a reasonable assumption $R_S \ll R_T$. Thus $\varepsilon = R_S/R_T$ is small. Then we have

$$\varepsilon\frac{dx}{d\tau} = (1-x) - \varepsilon A|x-y|x, \tag{14}$$

$$\frac{dy}{d\tau} = \zeta - (1 + A|x-y|)y. \tag{15}$$

As mentioned in [11, 12], Tihonvon and Fenichel's theorem is applied here to reduce the dynamics to the attracting close surface $x = 1+o(\varepsilon)$. Then we obtain the leading order equation of y

$$\frac{dy}{d\tau} = -(1 + A|1-y|)y + \zeta. \tag{16}$$

For convenience, from now on we will still use t to denote τ. Due to the fluctuating external salinity inputs or the fluctuations in the freshwater flux, ζ is varying following S^a and can be parameterized as mean part, denoted by $\bar{\zeta}$, and a noisy fluctuating process dL_t/dt. We thus have the following stochastic differential equation (SDE)

$$dy_t = f(y_t)dt + dL_t, \tag{17}$$

where $f(y) = -(1 + A|1-y|)y + \bar{\zeta}$.

For this stochastic Stommel model of THC, we use model parameters as estimated by Marotzke, as shown in Table 1.

Furthermore, A is calculated as a constant parameter, $A \simeq 5$, and $\bar{\zeta}(s) \simeq 1.018$. The stochastic equation above is now

$$dy_t = [-(1 + 5|1-y|)y + 1.018]dt + dL_t.$$

In fact we will consider an important case of a symmetric $\alpha-$stable Lévy motion L_t^α (see definition in the next section):

$$dy_t = [-(1 + 5|1-y|)y + 1.018]dt + dL_t^\alpha. \tag{18}$$

To quantify the impact of non-Gaussian noise on the box model, we will discuss the mean exit time (MET) for the stochastic system (18) for the rest of this paper.

TABLE 1. The parameters of box model. These parameters were estimated by Marotzke in 1996.

Parameter	Symbol	Value
Box volume	V	$1\times10^{14}m^3$
Temperature rebound coefficient	C_T	$3.17\times10^{-6}m^3s^{-1}$
Salinity rebound coefficient	C_S	$1.90\times10^{-6}m^3s^{-1}$
Temperature dilatancy coefficient	α_T	$1.5\times10^{-4}K^{-1}$
Salinity dilatancy coefficient	α_S	$8\times10^{-4}K^{-1}$
Water transport coefficient	γ	$1\times10^{9}m^3s^{-1}$
Heat rate of equatorial	T_e^a	$30°C$
Heat rate of polar	T_p^a	$0°C$

3. **Numerical schemes of mean exit time from a bounded domain.** In this section, we consider mean exit time (MET) of the stochastic Stommel model (18) with a symmetric α-stable Lévy motion L_t^α, with the generating triplet $(0, d, \nu_\alpha)$. The drift parameter θ is taken to be zero as it may be absorbed into the deterministic vector field, variance $d \geqslant 0$ and the jump measure $\nu_\alpha(dz) = C_\alpha \frac{dz}{|z|^{1+\alpha}}$, where $C_\alpha = \frac{\alpha}{2^{1-\alpha}\sqrt{\pi}}\frac{\Gamma(\frac{1+\alpha}{2})}{\Gamma(1-\frac{\alpha}{2})}$ for $0 < \alpha < 2$. A few authors have studied asymptotic mean exit time for stochastic systems with small α-stable Lévy noise [14, 16]. But here we consider a numerical approach for MET in the case of noise of any magnitude.

Given an open bounded interval D, the mean exit time for a solution process of SDE (1) starting at $y \in D$ is defined as

$$u(y) \triangleq \mathbb{E}\inf\{t \geq 0 : \ y_t(\omega) \notin D\}.$$

As in [10, 18], the mean exit time $u(y)$ satisfies the following differential-integral equation:

$$Au(y) = -1, \qquad y \in D, \tag{19}$$

$$u = 0, \qquad y \in D^c, \tag{20}$$

where the generator A is

$$Au = f(y)u'(y) + \frac{d}{2}u''(y) + \int_{\mathbb{R}\setminus\{0\}}[u(y+z) - u(y) - I_{\{|z|<1\}}zu'(y))]\nu_\alpha(dz) \tag{21}$$

and $D^c = \mathbb{R} \setminus D$ is the complement set of D.

Then Eq. (21) becomes

$$\frac{d}{2}u''(y) + f(y)u'(y) + \varepsilon C_\alpha \int_{\mathbb{R}\setminus\{0\}} \frac{u(y+z) - u(y) - I_{\{|z|<1\}}zu'(y))}{|z|^{1+\alpha}}dz \tag{22}$$

for $y \in D$ and $u = 0$ for $y \in D^c$.

We choose $D = (a, b)$ with $a > 0$ since $y > 0$. Translating Eq. (22) to a slightly different form: for $y \in (a, b)$

$$\frac{d}{2}u''(y) + f(y)u'(y) + \varepsilon C_\alpha \int_{\mathbb{R}\setminus\{0\}} \frac{u(y+z) - u(y) - I_{\{|z|<\delta\}}zu'(y))}{|z|^{1+\alpha}}dz = -1 \tag{23}$$

and for $y \notin (a,b), u(y) = 0$.
The positive value of δ in (23) is chosen to depend on the value of y.
The following numerical scheme follows [18]. Because u vanishes outside (a,b), Eq. (23) can be simplified by writing $\int_{\mathbb{R}} = \int_{-\infty}^{a-y} + \int_{a-y}^{b-y} + \int_{b-y}^{+\infty}$,

$$\frac{d}{2}u''(y) + f(y)u'(y) + \frac{\varepsilon C_\alpha}{\alpha}[\frac{1}{(a-y)^\alpha} + \frac{1}{(b-y)^\alpha}]u(y) + \varepsilon C_\alpha \int_{a-y}^{b-y} \frac{u(y+z) - u(y) - I_{\{|z|<\delta\}} z u'(y))}{|z|^{1+\alpha}} dz = -1 \tag{24}$$

for $y \in (a,b)$; and $u(y) = 0$ for $y \notin (a,b)$.
Since u is not smooth at the boundary points $y = a, b$, we rewrite Eq. (24) for ensuring the smooth integrand as

$$\frac{d}{2}u''(y) + f(y)u'(y) + \frac{1}{\alpha}[\frac{1}{(a-y)^\alpha} + \frac{1}{(b-y)^\alpha}]u(y) + \varepsilon C_\alpha \int_{a-y}^{a+y} \frac{u(y+z) - u(y)}{|z|^{1+\alpha}} dy + \varepsilon C_\alpha \int_{a+y}^{b+y} \frac{u(y+z) - u(y) - I_{\{|z|<\delta\}} z u'(y))}{|z|^{1+\alpha}} dz = -1 \tag{25}$$

for $y > \frac{b-a}{2}$, and

$$\frac{d}{2}u''(y) + f(y)u'(y) + \frac{1}{\alpha}[\frac{1}{(a-y)^\alpha} + \frac{1}{(b-y)^\alpha}]u(y) + \varepsilon C_\alpha \int_{b+y}^{b-y} \frac{u(y+z) - u(y)}{|z|^{1+\alpha}} dy + \varepsilon C_\alpha \int_{a-y}^{b+y} \frac{u(y+z) - u(y) - I_{\{|z|<\delta\}} z u'(y))}{|z|^{1+\alpha}} dz = -1 \tag{26}$$

for $0 < y < \frac{b-a}{2}$. We choose $\delta = min\{|a-x|, |b-x|\}$.
Let us divide the interval [(3a-b)/2, (3b-a)/2] into $2(b-a)J$ subintervals and define $y_j = jh$ for $\frac{3a-b}{2}J \leq j \leq \frac{3b-a}{2}J$ integer with $h = 1/J$. We denote the numerical solution of u at y_j by U_j and discretize the Eq. (25) with central difference for derivatives and "punched- hole" trapezoidal rule

$$\frac{U_{j-1} - 2U_j + U_{j+1}}{h^2}\frac{d}{2} + f(y_j)\frac{U_{j+1} - U_{j-1}}{2h} + \frac{\varepsilon C_\alpha}{\alpha}[\frac{1}{(a-y_j)^\alpha} + \frac{1}{(b-y_j)^\alpha}]U_j + \varepsilon C_\alpha h[\sum_{k=aJ-j}^{aJ+j}{}'' \frac{U_{j+k} - U_j}{|y_k|^{1+\alpha}} + \varepsilon C_\alpha h \sum_{k=aJ+j}^{bJ-j}{}'' \frac{U_{j+k} - U_j - (U_{j+1} - U_{j-1}) y_k/2h}{|y_k|^{1+\alpha}}] = -1 \tag{27}$$

where $j = ((b-a)/2)J, ((b-a)/2)J+1, ..., ((3b-a)/2)J-1$. The modified summation symbol $\sum''$ means that the quantities corresponding to the two end summation indices are multiplied by 1/2.

$$\frac{U_{j-1} - 2U_j + U_{j+1}}{h^2}\frac{d}{2} + f(y)\frac{U_{j+1} - U_{j-1}}{2h} + \frac{\varepsilon C_\alpha}{\alpha}[\frac{1}{(a-y_j)^\alpha} + \frac{1}{(b-y_j)^\alpha}]U_j + \varepsilon C_\alpha h[\sum_{k=bJ+j}^{bJ-j}{}'' \frac{U_{j+k} - U_j}{|y_k|^{1+\alpha}} + \varepsilon C_\alpha h \sum_{k=aJ-j}^{bJ+j}{}'' \frac{U_{j+k} - U_j - (U_{j+1} - U_{j-1}) y_k/2h}{|y_k|^{1+\alpha}}] = -1 \tag{28}$$

where $j = ((3a-b)/2)J+1, ..., ((b-a)/2)J-2, ((b-a)/2)J-1$. The bounded condition is the value of U_j vanish if the index $|j| \geq J$.

4. **Results and discussions.**

Now we consider the mean exit time of the stochastic Stommel model (18) proposed in Section 2 from various domain D and for various α values.

Note that in the model (18), $y \sim S_e - S_p$, i.e., y is proportional to the salinity (or density) difference between equatorial and polar regions in the oceans. Large y values indicate large density differences. It is the density difference that drives the thermohaline circulation. So large y values could lead to enhanced thermohaline circulation. The salinity difference y evolves with time t. The mean exit time $u(y)$, for $y \in D = (a, b)$, is the expected duration of time that the initial salinity difference y will remain in the bounded range $D = (a, b)$.

Case 1: $D = (0.3, 0.9)$.

First, we consider the salinity difference $y < 1$ and take $D = (0.3, 0.9)$. Figures 2–4 display the MET $u(y)$ for various α values. When $\alpha = 0.1$, the peak of MET is about $u = 0.0238$ at $y \approx 0.4$. This says that the initial salinity difference 0.4 will remain the longest time in the salinity difference range $D = (0.3, 0.9)$. The peak of $u(y)$ increases with α, and reaches the largest value around $\alpha = 1.0$, and then decreases gradually for $\alpha > 1$. Note that it takes longer time to escape from $(0.35, 0.45)$ for every α. This indicates that the salinity difference tends to remain within the range $(0.35, 0.45)$ for a longer time. For convenience, we say this is a "salinity-stability" or say the salinity difference range $(0.35, 0.45)$ is salinity-stable. This stability is with respective to the ambient domain D.

Case 2: $D = (1.3, 1.9)$.

When the salinity difference $y > 1$, how about the situation of $u(y)$, MET from the domain $D = (1.3, 1.9)$? The MET for various α values are in Figures 5–7. The salinity difference range $(1.35, 1.4)$ is salinity-stable, i.e., the salinity difference in this range remain so for a longer time. Comparing with Case 1, the peak values of u are smaller, although the domain D have the same length in both cases.

From Case 1 and Case 2, we see that there is a stable salinity difference range for either $y < 1$ and $y > 1$.

Case 3: $D = (0.4, 1.6)$.

Finally, we take the domain $D = (0.4, 1.6)$ which covers both salinity differences $y < 1$ and $y > 1$. Note that the term $|1-y|$ in (18) depends on $y > 1$ or $0 < y < 1$. Figures 8–11 show the MET for various α values. Observe that the peak value of MET increases with α. For all these tested α values, MET increases initially and then decreases, and the turning point is near $\{y = 1\}$. The $y = 1$ is more stable comparing with other salinity difference levels. In most of the domain $D = (0.4, 1.6)$, MET is relatively flat, indicating a low variability in salinity difference in the range $0.4 < y < 1.6$.

In summary, we have considered a simple stochastic box model for THC. It is described by a SDE with a α-stable Lévy motion. The α values, $0 < \alpha < 2$, may be regarded as the index of non-Gaussianity. When $\alpha = 2$, the α-stable Lévy motion becomes the usual (Gaussian) Brownian motion. We quantity some dynamical features of this stochastic model by computing the mean exit time for various α values and for different domain D. We have observed that some salinity difference levels remain in certain ranges for longer times than other salinity difference levels, for different α values. This indicates a lower variability for these salinity difference levels. Realizing that it is the salinity differences that drive the thermohaline circulation, this lower variability could mean a stable circulation, which may have further implications for the global climate dynamics.

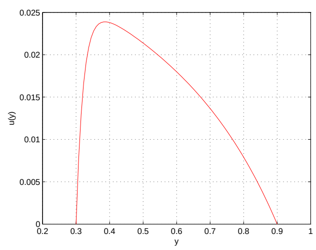

FIGURE 2. Mean exit time $u(y)$ for $dy_t = [-(1+5|1-y|)y + 1.018]dt + dL_t^\alpha$ at $\alpha = 0.1$, escaping from $D = (0.3, 0.9)$.

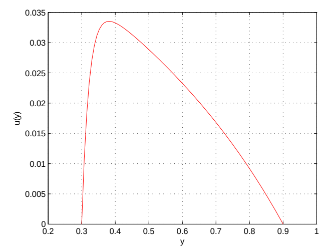

FIGURE 3. Mean exit time $u(y)$ for $dy_t = [-(1+5|1-y|)y + 1.018]dt + dL_t^\alpha$ at $\alpha = 1.0$, escaping from $D = (0.3, 0.9)$.

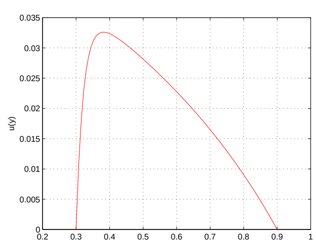

FIGURE 4. Mean exit time $u(y)$ for $dy_t = [-(1+5|1-y|)y + 1.018]dt + dL_t^\alpha$ at $\alpha = 1.9$, escaping from $D = (0.3, 0.9)$.

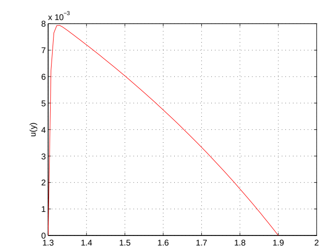

FIGURE 5. Mean exit time $u(y)$ for $dy_t = [-(1+5|1-y|)y + 1.018]dt + dL_t^\alpha$ at $\alpha = 0.1$, escaping from $D = (1.3, 1.9)$.

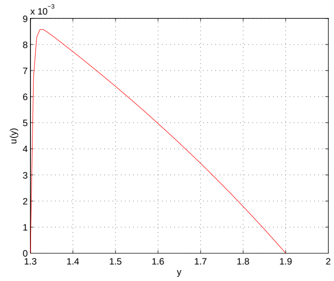

FIGURE 6. Mean exit time $u(y)$ for $dy_t = [-(1+5|1-y|)y + 1.018]dt + dL_t^\alpha$ at $\alpha = 1.0$, escaping from $D = (1.3, 1.9)$.

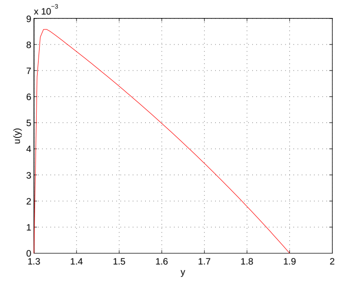

FIGURE 7. Mean exit time $u(y)$ for $dy_t = [-(1+5|1-y|)y + 1.018]dt + dL_t^\alpha$ at $\alpha = 1.9$, escaping from $D = (1.3, 1.9)$.

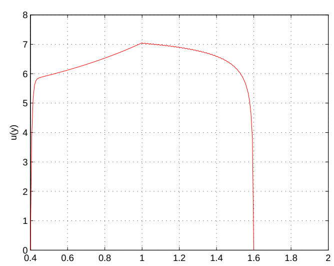

FIGURE 8. Mean exit time $u(y)$ for $dy_t = [-(1+5|1-y|)y + 1.018]dt + dL_t^\alpha$ at $\alpha = 0.1$, escaping from $D = (0.4, 1.6)$.

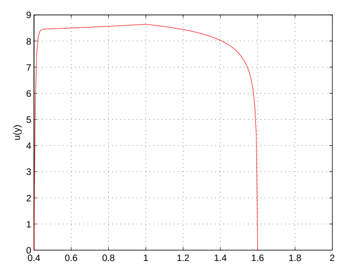

FIGURE 9. Mean exit time $u(y)$ for $dy_t = [-(1+5|1-y|)y + 1.018]dt + dL_t^\alpha$ at $\alpha = 0.4$, escaping from $D = (0.4, 1.6)$.

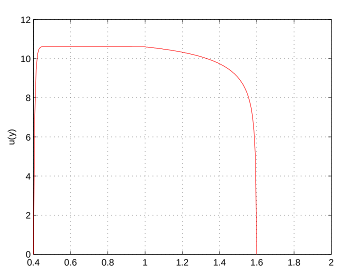

FIGURE 10. Mean exit time $u(y)$ for $dy_t = [-(1+5|1-y|)y + 1.018]dt + dL_t^\alpha$ at $\alpha = 1.0$, escaping from $D = (0.4, 1.6)$.

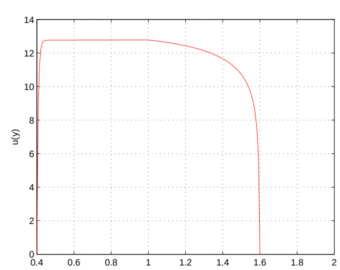

FIGURE 11. Mean exit time $u(y)$ for $dy_t = [-(1+5|1-y|)y + 1.018]dt + dL_t^\alpha$ at $\alpha = 1.5$, escaping from $D = (0.4, 1.6)$.

REFERENCES

[1] H. Stommel, *Thermohaline convection with two stable regimes of flow*, Tellus., **13** (1961), 224–230.

[2] L. Mass, *A simple model for the three-dimensional, thermally and wind-driven ocean circulation*, Tellus., **46** (1994), 671–680.

[3] S. Rahmstorf, *On the freshwater forcing and transport of the Atlantic thermohaline circulation*, Clim. Dyn., **12** (1996), 799–811.

[4] J. Scott, J. Marotzke and P. Stone, *Interhemispheric thermohaline circulation in a coupled box model*, J. Phys. Oceanogr., **29** (1999), 351–365.

[5] P. Cessi, *A simple box model of stochastically-forced thermohaline flow*, J. Phys. Oceanogr., **24** (1994), 1911–1920.

[6] S. M. Griffies and E. Tziperman, *A linear thermohaline oscillator driven by stochastic atmospheric forcing*, J. Climate, **8** (1995), 2440–2453.

[7] G. Lohmann and J. Schneider, *Dynamics and predictability of Stommel's box model: A phase space perspective with implications for decadal climate variability*, Tellus., **51** (1998), 326–336.

[8] D. Applebaum, "Lévy Processes and Stochastic Calculus," Cambridge Studies in Advanced Mathematics, **93**, Cambridge University Press, Cambridge, 2004.

[9] K.-I. Sato, "Lévy Processes and Infinitely Divisible Distributions," Cambridge Studies in Advanced Mathematics, **68**, Cambridge University Press, Cambridge, 1999.

[10] J. Brannan, J. Duan and V. Ervin, *Escape probability, mean residence time and geophysical fluid particle dynamics*, Physica D, **133** (1999), 23–33.

[11] P. Glendinning, *View from the pennines: Box models of the oceanic conveyor belt*, Mathematics Today (Southend-on-Sea), **45** (2009), 230–232.

[12] N. Berglund and B. Gentz, *Metastability in simple climate models: Pathwise analysis of slowly driven langevin equations*, Stochastics and Dynamics, **2** (2003), 327–356.

[13] J. Marotzke, *Analysis of thermohaline feedbacks*, in "Decadal Climate Variability: Dynamics and Predictability," (eds. D. L. T. Anderson and J. Willebrand), Springer-Verlag, 1996.

[14] P. Imkeller and I. Pavlyukevich, *First exit time of SDEs driven by stable Lévy processes*, Stoch. proc. Appl., **116** (2006), 611–642.

[15] W. A. Woyczyński, *Lévy processes in the physical sciences*, in "Lévy Processes" (eds. O. E. Barndorff-Nielsen, T. Mikosch and S. I. Resnick), Birkhäuser Boston, Boston, MA, (2001), 241–266.

[16] Z. Yang and J. Duan, *An intermediate regime for exit phenomena driven by non-Gaussian Lévy noises*, Stochastics and Dynamics, **8** (2008), 583–591.

[17] H. Dijkstra, "Nonlinear Physical Oceanography," 2^{nd} edition, Springer, 2005.

[18] T. Gao, J. Duan, X. Li and R. Song, *Mean exit time and escape probability for dynamical systems driven by Lévy noise*, submitted, 2011.

Received November 2011; revised January 2012.

E-mail address: xjwang@mail.hust.edu.cn
E-mail address: jhwen86@gmail.com
E-mail address: ljp@lasg.iap.ac.cn
E-mail address: duan@iit.edu